4.4b Modeling and Optimization

Designing a can

You have been asked to design a one-liter oil can shaped like a right-circular cylinder. What dimensions will use the least material?

$V = 1\ell = 1000\ cm^3 = \pi r^2 h$

$h = \frac{1000}{\pi r^2}$

min S = surface Area

$S = 2\pi r^2 + 2\pi r h$

$S = 2\pi r^2 + 2\pi r \cdot \frac{1000}{\pi r^2}$

$S = 2\pi r^2 + \frac{2000}{r}$

$S' = 4\pi r - \frac{2000}{r^2} = 0$

min at $r = 5.41926$

$S'' = 4\pi + \frac{4000}{r^3}$ $S''(5.41926) > 0$

$h = \frac{1000}{\pi (5.41926)^2} = 10.8385$

$h = 2r$

$h = d$

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A cylindrical can holds a fixed volume. Find the ratio of height to radius for the can with least surface area.

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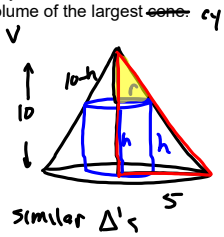
A cylinder is inscribed in a cone of height 10 and radius 5. Find the volume of the largest ~~cone~~ cylinder



similar Δ's

$V = \pi r^2 h$

$\frac{r}{5} = \frac{10-h}{10}$ $r = \frac{10-h}{2}$

$V = \pi \left(\frac{10-h}{2}\right)^2 \cdot h$

$V' = \frac{\pi (h-10)(3h-10)}{4} = 0$

$h = 10$ $V = 0$

$h = 10/3$ $V = \frac{1000\pi}{27}$

$h = 0$ $V = 0$

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